

Lecture 10 - Work, Power and Energy I

Introduction

Work, power and energy are all terms which are very familiar to us. The colloquial use of these terms is very close to their technical use in mechanics. In fact you probably already know that the unit of power is the watt (W), though you may be most familiar with it from its use on lightbulbs. (the symbol W is also used to indicate work so be careful to interpret this symbol correctly from its context) Nevertheless the unit of power is the watt regardless of the origin of the power. However we have to make our definitions of mechanical work, power and energy precise. Lets get started by considering work.

The dot product of two vectors

Before we start, we need to understand a new product operation which can be carried out with vectors. So far, we learned how to multiply a scalar (like time) times a vector (like velocity). Now we need to use a new product called a dot product. For example if I take the dot product of the force and the displacement, in two dimensions, we have,

$$\vec{F} \cdot \Delta\vec{r} = F_x\Delta x + F_y\Delta y \quad (1)$$

Note that the dot product of two vectors produces a scalar. This is a nice way to get rid of vectors! In polar co-ordinates, we have,

$$\vec{F} \cdot \Delta\vec{r} = F\Delta r\cos(\theta) \quad (2)$$

where F is the magnitude of the force and Δr is the magnitude of the displacement. The angle θ is the angle between the two vectors. Note that the order of the vectors in the dot product does not matter. You can also think of this dot product as the component of the force in the direction of the displacement or visa-versa.

Work

For small displacements, the mechanical work is defined as the dot product of the force vector \vec{F} times the displacement vector $\Delta\vec{r}$, that is,

$$\Delta W = \vec{F} \cdot \Delta\vec{r} \quad (3)$$

From this expression we see that the units of work are given by,

$$[\Delta W] = [\vec{F}][\Delta\vec{r}] = Nm \quad (4)$$

However work is so important that a new unit is defined, the Joule ($J = Nm$). If the force is a constant, then equation (3) is true for even large displacements. However in general we have,

$$W = \sum_i \vec{F}_i \cdot \Delta\vec{r}_i \quad (5)$$

This looks complicated, but we will do special cases that are not as nasty. However before doing examples lets look at the consequences of doing work.

What happens when we do work?

- (i) Starting a bike from rest (leads to acceleration or “kinetic” energy).
- (ii) Pushing a block across a table against friction (leads to “dissipation”).
- (iii) Lifting a block against gravity(leads to stored or “potential” energy).

Friction leads to dissipation or energy loss and is called a non-conservative force. Gravity is a conservative force as when we do work against gravity, the energy is stored. No energy is lost unless there is friction or drag. Moving objects also have stored energy and that energy is called kinetic energy.

Figuring out the relations between energy and work

Potential energy

When we lift a mass m a height h , we must do work against the weight of the object, $w = mg$. Since the weight is a constant and the force is in the same direction as the displacement, the work done in lifting straight up is,

$$W = \vec{F} \cdot \vec{r} = mgh\text{Cos}(0) = mgh \quad (6)$$

This work is stored, so we define the potential energy gained by a mass lifted through height h to be,

$$PE = mgh = mgy_f - mgy_i = PE_f - PE_i \quad (7)$$

The latter two expressions are the more formal statement of the change in the potential energy(PE) as the final potential energy minus the initial potential

energy. Later we shall also figure out the potential energy stored in a spring. In general there are many ways to store energy, for example in batteries, in capacitors, in hydrogen fuel cells etc. Nevertheless it would be really helpful to find better ways to store energy efficiently in order to take advantage of alternative energy sources such as solar energy. At the moment batteries are the best, but they are expensive and actually not good enough for many applications such as electric car applications.

Now consider pushing a block up an inclined plane. Do we do the same amount of work? Assume the inclined plane is frictionless.

$$W = \vec{F} \cdot \vec{r} = mgsin(\theta) \frac{h}{sin(\theta)} = mgh \quad (8)$$

It's the same. In fact it does not matter what path we take up the hill, as long as there is no friction. This is a special property. Some forces have it, some don't. Forces that don't waste any energy are called conservative forces and gravity is an example. These forces can be used to store energy. Other forces, such as friction or drag, dissipate energy and lead to inefficiency. Note however that sometimes we want to waste energy, such as when we use the brakes in a car where the brake pads provide the friction force that slows the car.

Converting work or stored energy to motion

In the bicycle example or if we drop a mass, work or stored energy is converted to motion. If there is no friction there is no energy loss so the amount of energy in the motion has to be the same as the stored energy or the work. In the case of constant acceleration, the amount of energy in motion at speed v is found from the constant acceleration kinematics formulae. Consider a mass m which is initially at rest on a skating rink, to which we apply a horizontal force of magnitude F . We assume that there is no friction, so the mass accelerates in the horizontal direction according to Newton's second law, $a = F/m$. The velocity can then be found from the constant acceleration formula,

$$v^2 = v_0^2 + 2a\Delta x = v_0^2 + 2\frac{F}{m}\Delta x \quad (9)$$

Now we multiply both sides by $m/2$ and find that,

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F\Delta x = Work \quad (10)$$

The left hand side is the energy stored in motion! The energy due to motion is called the kinetic energy, (KE) and we write the equation above as

$$KE_f - KE_i = Work \quad (11)$$

where $KE = mv^2/2$.

Converting work to energy

The examples above describe what happens when we convert work to potential energy, or to kinetic energy or we dissipate the energy. In general a combination of these processes occur. The problems in Set 4 challenge you to consider various way work can lead to different forms of energy or energy dissipation and we will discuss various cases in the following lectures.